Calculations for ZolarDisc Spacecraft

Radiation

$$H(d)=H_0 e^{-\ln(2)\frac{d}{d_0}} \rightarrow d_1=d_0\frac{\ln(\frac{H_1}{H_2})}{-\ln(2)}=1.5\,m$$

$H_0=900\frac{mSV}{a}$ initial radiation dose rate for interplanetary space

$H_1=2.4\frac{mSV}{a}$ desired radiation dose rate (average on Earth)

d thickness of shielding

$d_0=18\,cm$ halfing thickness of shield (water)

d_1 thickness of shielding to achieve reduction from H_0 to H_1

Artificial Gravity

$$g=r\frac{4\pi^2}{T^2} \rightarrow T=\sqrt{r\frac{4\pi^2}{g}}=31.7\,s \rightarrow v=\frac{1}{T}=1.89\frac{1}{min}$$

$g=9.81\frac{m}{s^2}$ earth gravitational acceleration at surface

$r=250\,m$ rotation radius

T rotation period

v rotation frequency

Solar Power

Data for solar cells from Wyrsch, Ultra-light amorphous silicon cell for space applications, 2006

Solar Cell Mass

$$m_{labcell}=A\rho=5.89\,t \rightarrow m_{additional\,wiring\text{ and }protection}\approx 10\,t$$

m mass of solar cells

$\rho = 30 \frac{g}{m^2}$ density of laboratory solar cell

$A = \pi r^2 = 196350\, m^2$ area of disc of solar modules

Electric Power

$$P_{Earth} = \frac{P}{A} A = 24\ MW$$

P_{Earth} maximum solar power at Earth

$\frac{P}{A} = 122.25 \frac{W}{m^2}$ power density of solar cell

$A = \pi r^2 = 196350\, m^2$ area of disc of solar modules

Power at Mars and Venus

$$P_{Mars,Venus} = P_{Earth} \frac{r_{Earth}^2}{r_{Mars,Venus}^2}$$

$P_{Mars,Venus}$ maximum solar power at Mars and Venus

P_{Earth} maximum solar power at Earth

$r_{Earth,Mars,Venus}$ semi-major axis of Earth, Mars and Venus

In-Orbit Propellant Collection

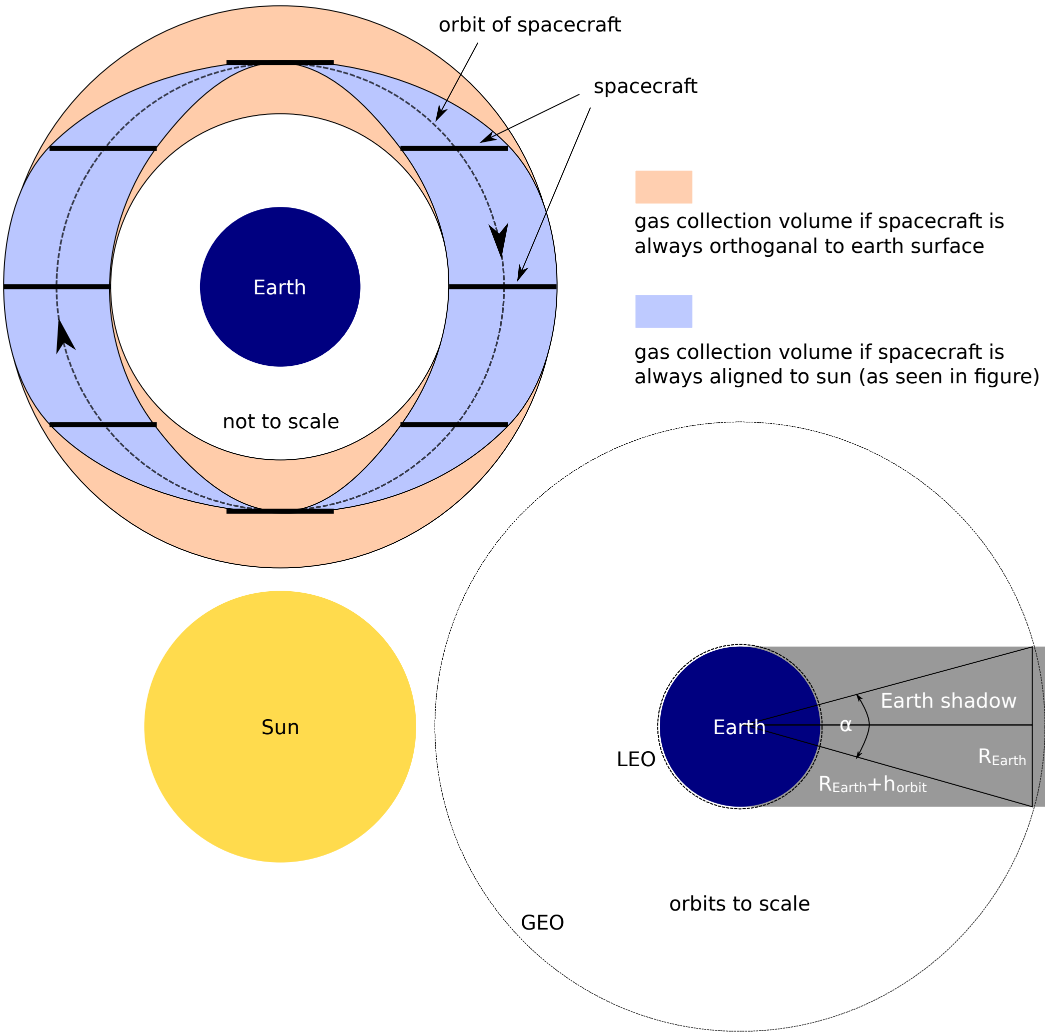

Fig. 1: Gas collection volume and influence of earth shadow for spacecraft in orbit

Gas Mass

collected gas mass per day in 350 km orbiting

$$m = V\rho = A_{collection}\, v\, t\, \rho = 2.3\, t$$

V gas volume swiped by the disc during one day. It is assumed that all gas in that volume is collected (collection efficiency of 1)

$\rho = 3.5 \cdot 10^{-11} \frac{kg}{m^3}$ air density at 350 km altitude

$A_{collection} = \pi \bar{r}_{collection}^{\,2} = \pi \frac{1}{\pi} \int_0^{\pi} (r \cdot \sin(x))^2 dx = 0.5\, \pi r^2 = 0.5\, A$ average collection area (s. Fig. 1). Spacecraft is kept align to sun for maximal power production

$A = \pi r^2 = 196350\, m^2$ area of disk of solar modules

$\mathrm{v} = 27720 \frac{km}{h}$ orbital speed

$t = 1\, d = 24\, h$ collection time (1 day)

Breaking Force

It is assumed that the breaking force is purely generated by having to accelerate the captured gas from average zero velocity to the orbital velocity of the spacecraft.

$$F = \dot{m}\, \mathrm{v} = \frac{m}{t} \mathrm{v} = 204\, N$$

F breaking force

$\dot{m}$ change of mass (time derivative)

$m = 2.3\, t$ accelerated gas mass per day in 350 km orbiting

$t = 1\, d = 24\, h$ acceleration time (1 day)

$\mathrm{v} = 27720 \frac{km}{h}$ orbital speed

Available Thrust

The fraction of time the spacecraft flies in the shadow of the earth can be derived from the geometry in Fig. 1 as:

$$frac = \frac{\alpha}{360°} = 2 \cdot \arcsin\left(\frac{R_{Earth}}{R_{Earth} + h_{orbit}}\right) \cdot \frac{1}{360°}$$

We therefore get:

$frac_{LEO} = 0.40$

$frac_{GEO} = 0.05$

with

$R_{Earth} = 6378\, km$

$h_{orbit,LEO} = 350\, km$

$h_{orbit,GEO} = 35786\, km$

The solar output is therefore reduced by 40% when in LEO. This reduction decreases with increasing orbit reaching 5% reduction when in GEO.

Available thrust in LEO can then be calculated as

$$F_{available}=\frac{F}{P}P_{available}=\frac{F}{P}0.6\,P_{Earth}=360\,N$$

$\frac{F}{P}=25\frac{N}{MW}$ power density of thruster: VASIMR, using Xenon as propellant. Oxygen as propellant might has worse performance but we also only need 204 N to compensate for breaking force. If needed, a higher orbit and slower gas collection rate could compensate for worse than expected thruster performance.

$P_{Earth}=24.MW$ maximum solar power at Earth

Propellant Consumption

$$F=I_{SP}\dot{m}\,g=I_{SP}\frac{m}{t}g \rightarrow m=\frac{F}{I_{SP}\,g}=0.7\,t$$

$F=204\,N$ thrust needed to compensate for breaking force

$I_{SP}=2500\,s$ specific impulse (VASIMR)

$g=9.81\frac{m}{s^2}$ earth gravitational acceleration at surface

$\dot{m}$ change of mass (time derivative)

$t=24\,h$ duration of thrust

m propellant mass used every day to sustain thrust

Collection Duration

The time for the spacecraft to collect $m_{propellant}=200\,t$ can no be expressed as

$$t_{propellant}=\frac{m_{propellent}}{\dot{m}_{collection}-\dot{m}_{consumption}}=128\,d$$

$\dot{m}_{collection}=2.3\frac{t}{d}$ gas mass collection rate

$\dot{m}_{consumption}=0.7\frac{t}{d}$ gas mass consumption rate

Propellant consumption for rotational acceleration of collected gas has not been considered. Real propellant consumption will therefore be slightly higher.

Power Demand to compress Gas

The power demand to compress the collected gas (assuming 50% efficiency) can be expressed as

$$P_{compression}=\frac{1}{\eta}\frac{W_{1\to 2}}{t}=\frac{1}{\eta}\frac{p_0 V_0}{t}\ln\left(\frac{p_0}{p_1}\right)=\frac{1}{\eta}\frac{p_0\frac{m_0}{\rho_0}}{t}\ln\left(\frac{p_0}{p_1}\right)=0.5\,MW$$

$W_{1\to 2}$ energy needed to compress gas

$\eta=0.5$ energy efficiency of compression process

$p_0=11.2\,10^{-6}\,Pa$ initial gas pressure

$p_1=500\,\mathrm{bar}$ final gas pressure

$\rho_0=3.5\cdot 10^{-11}\frac{kg}{m^3}$ initial gas density

$V_0=\frac{m_0}{\rho_0}$ initial gas volume

$m_0=2.3\,t$ mass of gas collected

$t=24\,h$ compression time (1 day)

Temperature of Disc

First we only consider solar radiation and the Stefan-Boltzman Law (http://en.wikipedia.org/wiki/Stefan%E2%80%93Boltzmann_law)

$$P_{absorption}=P_{emission}$$

$$P_{absorption}=(1-\eta)\,P_{Sun,LEO}=(1-\eta)\,I_{Sun,LEO}A_0$$

$$P_{emission}=2\,A_0\,\sigma\,\varepsilon\,T_1^4$$

The average temperature of the disc will therefore be

$$\overline{T_1}=\sqrt[4]{\frac{(1-\eta)\,\overline{I_{Sun,LEO}}\,A_0}{2\,A_0\,\sigma}}=283\,K=10\,°C$$

and the maximum temperature (on the day side):

$$\hat{T}_1=\sqrt[4]{\frac{(1-\eta)\hat{I}_{Sun,LEO}A_0}{2A_0\sigma}}=322\,K=48\,°C$$

with

$\eta=0.1$ efficiency of solar modules

$I_{Sun,LEO}$ solar irradiance in LEO

$\hat{I}_{Sun,LEO}=1350\frac{W}{m^2}$ maximum solar irradiance in LEO (day side)

$\overline{I_{Sun,LEO}}=0.6\cdot1350\frac{W}{m^2}=810\frac{W}{m^2}$ average solar irradiance in LEO (day and night side, see "Available Thrust")

$A_0=19635\,m^2$ (PV) disc area

$\varepsilon\approx1$ emissivity of the solar modules

$\sigma=5.67\cdot10^{-8}\,Wm^{-2}K^{-4}$ Stefan-Boltzman constant

We now consider also the energy transfer due do thermal energy of the gas and the bonding energy that is freed when the gas atoms form molecules. We assume that this effects mainly the rim of the disc where gas density is much higher than in the center of the disc. We assume a 0.5 m wide stripe at the disc rim acting as a heat radiator. Its temperature T_2 can be derived as follows

$$P_{Sun,LEO,rim}+P_{O_2}+P_{thermal}=P_{emission,rim}=2A_2\sigma T_2^4\rightarrow T_2=\sqrt[4]{\frac{P_{Sun,LEO,rim}+P_{O_2}+P_{thermal}}{2A_2\sigma}}$$

$P_{O_2}=\frac{P}{m}\dot{m}=0.41\,MW$ power due to recombination of gas atoms to molecules

$\frac{P}{m}=15.5\frac{MJ}{kg}$ bonding energy of oxygen

$\dot{m}=2.3\frac{t}{d}$ mass of atomic oxygen (gas) bonding to molecules (we assume that all gas particles are either oxygen or have a similar bonding energy)

$A_2\approx0.5\,m\cdot2\pi\,250\,m=393\,m^2$ area of radiator at disc rim

$P_{thermal}=\frac{3}{2}\dot{N}k_B T_0=0.03\,MW$ power input by cooling all collected gas to 0 K (upper limit, actual power will be lower)

$\dot{N}=\frac{M}{mt}$ number of gas particles cooled down per day

$M=17.1\frac{g}{mol}$ molar mass of gas in 350 km orbit

$m=2.3\,t$ gas cooled down per day

$t=1\,d$

$P_{Sun,LEO,rim}=I_{Sun,LEO}A_2$

We get an average temperature of

$$\overline{T}_2=\sqrt[4]{\frac{\overline{I_{Sun,LEO}}A_2+P_{O_2}+P_{thermal}}{2A_2\sigma}}=331\,K=58\,°C$$

and the maximal Temperature at the day side of the orbit:

$$\hat{T}_2=\sqrt[4]{\frac{\hat{I}_{Sun,LEO}A_2+P_{O_2}+P_{thermal}}{2A_2\sigma}}=360\,K=87\,°C$$